BROWNIAN DIFFUSION IN NONEQUILIBRIUM GAS

O. A. Grechannyi

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The Brownian motion in a nonequilibrium gas is considered within the framework of the stochastic transfer theory in linear Boltzmann systems. The equation for Brownian diffusion of particles is derived in inhomogeneous flows of rarefied gas.

Some general approaches for investigating Brownian motion of particles in a nonequilibrium thermostatic regulator were developed in [1-4]. When analyzing the motion of a mirror-reflecting sphere in gas Brownian diffusion was studied in [4] in an inhomogeneous flow of rarefied gas which is based on a modified Markov method. However, in the case of gas thermostatic regulator it would be more to the point to investigate Brownian diffusion in the framework of kinetic theory. Such analysis was carried out in [5] for the gas equilibrium case.

In the present article Brownian motion of particles in a nonequilibrium gaseous medium is again considered on the basis of kinetic theory. The results of the analysis are identical in corresponding situations with the results given in [4]; consequently, they provide a kinetic basis for the method as given in [4]. The obtained expressions for the tensor of diffusion and of dynamic friction enable one to write down the Langevin equations for Brownian motion in an inhomogeneous anisotropic medium; by employing these expressions the equation of Brownian diffusion can be obtained which takes into account the effect of the inertia of a particle on its motion in the coordinate space. To give an example the stress-bearing effect is studied on the stationary particle distribution in Poiseuille flow and in the gravitational field. It results in an inhomogeneous particle distribution of a flow section and an additional pulling down in the direction of the gravitational forces.

1. The motion is considered of a system of particles of mass m and mean density n in an inhomogeneous gaseous medium with density n_0 and molecule mass m_0. Under the assumption that $n_0 \gg n$, the interaction between the particles can be ignored if one compares it with their interaction with gas molecules. It is also assumed by us that the distribution function of the molecules is independent of the state of particle m since the change in f due to interaction between the molecules of the medium and the particles is proportional to n. These approximations (in the case of interaction between the particles and gas molecules due to collision) are equivalent to an assumption that the evolution of the distribution function F of particles m has a Markovian character and enables one to analyze this evolution using stochastic theory of transport phenomena in linear Boltzmann systems of Tolubinskii [6] in which the particle velocity is approximated by a Markov jump-like process.

In describing the interaction between the particles m and the gas molecules we shall only consider the potential of rigid spheres. The same characteristics of a Markov jump-like process, namely the collision frequency ν and the probability density of transfer in a unit of time W are given in this case by

$$\nu(t, \vec{r}, \vec{v}) = \sigma \int d\vec{v}_0 g f(t, \vec{r}, \vec{v}_0), \tag{1}$$

$$W(t, \vec{r}, \vec{v} \mid \vec{v}') = \sigma \int d\vec{v}_0 \int d\vec{v}_0' \, \delta(\vec{v}_s - \vec{v}_s') \, \delta\left(\frac{g^2 - g'^2}{2}\right) f(t, \vec{r}, \vec{v}_0'). \tag{2}$$

The above reflects the laws of conservation of energy and of momentum in collisions and the following notation is used: $\vec{v}_s = (m\vec{v} + m_0\vec{v}_0)/(m + m_0)$, $\vec{g} = \vec{v} - \vec{v}_0 = \vec{e}g$. The equation for F is given by [6]

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$$\frac{\partial F}{\partial t}+\vec{v}\cdot\frac{\partial F}{\partial \vec{r}}+\vec{E}\cdot\frac{\partial F}{\partial \vec{v}}=\int d\vec{v}'W(\vec{v}\,|\,\vec{v}')F(t,\vec{r},\vec{v}')-\nu F \tag{3}$$

which can be regarded as generalized Fokker–Planck equations for the case of particle motion in nonequilibrium gaseous medium for any value of the ratio $\varepsilon = m_0/m$.

2. Brownian motion occurs if $\varepsilon \ll 1$. It is considered by us on the basis of (3). Since the change of the particle velocity when colliding is given by $\Delta\vec{v} = [\varepsilon/(1+\varepsilon)](\vec{g}+\vec{e}'g) = -\varepsilon\Delta\vec{v}_0$, therefore by setting $\Delta v_0 \sim 1$, one obtains $\Delta v \sim \varepsilon$. The integral of Fokker–Planck collisions follows from the Boltzmann collision integral in (3) if $F(t, \vec{r}, \vec{v}')$ is expanded into a series of $\Delta\vec{v}$ and the first two nonvanishing terms are retained:

$$\hat{J}_c F=\frac{\partial}{\partial \vec{v}}\cdot\vec{A}(t,\vec{r},\vec{v})F(t,\vec{r},\vec{v})+\frac{\partial^2}{\partial \vec{v}^2}:\underset{\rightarrow}{D}_v(t,\vec{r},\vec{v})F(t,\vec{r},\vec{v}), \tag{4}$$

and hence by using (2)

$$\vec{A}=\sigma\int d\vec{v}_0\int d\vec{e}'g\Delta\vec{v}f(t,\vec{r},\vec{v}_0+\Delta\vec{v}_0), \tag{5}$$

$$\underset{\rightarrow}{D}_v=\frac{1}{2}\sigma\int d\vec{v}_0\int d\vec{e}'g\Delta\vec{v}\Delta\vec{v}f(t,\vec{r},\vec{v}_0+\Delta\vec{v}_0). \tag{6}$$

From the physicist's point of view it is interesting to study the approximation of the state of a Brownian particle in equilibrium to the state of the surrounding gaseous medium when $|\vec{s}| = |\vec{v}-\vec{u}(\vec{r})| \sim [\varepsilon kT(\vec{r})]^{1/2}$ (the particle energy in the external field $\vec{E}$ is assumed to be small compared with the average thermal energy). By assuming that in this stage one has $|\vec{c}| = |\vec{v}_0-\vec{u}(\vec{r})| \sim 1$, one obtains that $|s| \sim \varepsilon^{1/2}$. Therefore, if the coefficients of $\vec{A}$ and $\underset{\rightarrow}{D}_V$ in (4) are calculated with an accuracy up to the terms of the order $\varepsilon^{3/2}$ or ε^2, respectively, then (4) takes into account the terms of the order of ε in the correct expansion of the Boltzmann collision integral in the unique series in powers of ε.

3. The calculation of the pulling down and diffusion coefficients in (4) is easily carried out in the variables $\vec{c}$ and $\vec{s}$. For a nonequilibrium distribution function of gas molecules its thirteen-moments approximation [7] is adopted

$$f=f_0\left[1-\underset{\rightarrow}{\tau}:\frac{\vec{c}\vec{c}}{2pV_0^2}-\vec{q}\cdot\frac{\vec{c}}{pV_0^2}\left(1-\frac{\vec{c}\cdot\vec{c}}{5V_0^2}\right)\right]. \tag{7}$$

In the above f_0 denotes the equilibrium distribution

$$f_0(t,\vec{r},\vec{v})=n_0(2\pi V_0^2)^{-3/2}\exp[-(\vec{v}_0-\vec{u})^2/2V_0^2], \tag{8}$$

the dot denoting here the scalar product, the colon the convolution operation of tensors, and the product of vectors $\vec{c}\vec{c}$ being the inner product.

In accordance with (8) $\vec{A}$ and $\underset{\rightarrow}{D}_V$ are represented by

$$\vec{A}=\vec{A}_0+\vec{A}_c+\vec{A}_T,\quad \underset{\rightarrow}{D}_v=\underset{\rightarrow}{D}_0+\underset{\rightarrow}{D}_c+\underset{\rightarrow}{D}_T, \tag{9}$$

where $\vec{A}_0$, $\underset{\rightarrow}{D}_0$ are transfer coefficients in a locally equilibrium medium; $\vec{A}_C$, $\underset{\rightarrow}{D}_C$ take into account the effects of tangential stresses in a gas flow; and $\vec{A}_T$, $\underset{\rightarrow}{D}_T$ the thermo-bearing effect. When evaluating $\vec{A}$ and $\underset{\rightarrow}{D}_V$ by using the formulas (5) and (6), it is necessary to expand in powers of ε all the quantities appearing there, and one ignores the terms of orders higher than $\varepsilon^{3/2}$ and ε^2, respectively. The calculations are quite straightforward, though cumbersome. They are omitted here, only the final results being given:

$$\vec{A}_0=\sigma\varepsilon\int d\vec{c}f_0(c)\int d\vec{e}'\left[\vec{c}c\frac{\vec{s}\cdot\vec{c}}{V_0^2}+c^3\vec{e}'\frac{\vec{s}\cdot\vec{e}'}{V_0^2}-c\vec{s}-\vec{c}\frac{\vec{s}\cdot\vec{c}}{c}\right]=\gamma\vec{s},$$
$$\underset{\rightarrow}{D}_0=\frac{1}{2}\sigma\varepsilon^2\int d\vec{c}f_0(c)\int d\vec{e}'c[\vec{c}\vec{c}+c^3\vec{e}'\vec{e}']=\gamma V_0^2\varepsilon I=D_v\underset{\rightarrow}{I}. \tag{10}$$

In the above the coefficient of dynamic friction in the locally equilibrium gas was introduced

$$\gamma=\frac{16}{3}\sqrt{\frac{\pi}{2}}\,a^2\varepsilon V_0 n_0 \tag{11}$$

and the diffusion coefficient $D_V = \gamma\varepsilon V_0^2$ in the velocity space. The expressions (10)-(11) are identical to the familiar results of the theory of Brownian motion in a homogeneous gas [5]. However, since in our case n_0 and T depend on $\vec{r}$, therefore $\gamma = \gamma(\vec{r})$ and $D_V = D_V(\vec{r})$,

$$\vec{A}_c = -\frac{\sigma\varepsilon}{2pV_0^2}\int d\vec{c}c^3 f_0(c)\int d\vec{e}'\left[c^2\vec{e}'\frac{\vec{s}\cdot\vec{e}'}{V_0^2}\underset{\to}{\tau}:\vec{e}'\vec{e}' - 2\vec{e}'\underset{\to}{\tau}:\vec{s}\vec{e}'\right] = -\frac{1}{5}\gamma\frac{\underset{\to}{\tau}\cdot\vec{s}}{p},$$
$$\underset{\to}{D}_c = -\frac{\sigma\varepsilon}{2pV_0^2}\int d\vec{c}c^5 f_0(c)\int d\vec{e}'\vec{e}'\vec{e}'\underset{\to}{\tau}:\vec{e}'\vec{e}' = -\frac{3}{5}D_v\frac{\underset{\to}{\tau}}{p}. \tag{12}$$

These expressions describe the so-called stress-bearing effect for Brownian motion of particles in an inhomogeneous gas flow which appears in the additional drift of a particle in the direction $\underset{\to}{\tau}\cdot\vec{s}$, as well as in the anisotropic addition (12) to the diffusion coefficient $\underset{\to}{D}_0$.

Finally, the heat-bearing effect is taken into account by the expressions

$$\vec{A}_T = -\frac{\sigma\varepsilon}{pV_0^2}\int dc f_0(c)\int d\vec{e}'c^3\left[\vec{e}'\frac{\vec{e}'\cdot\vec{q}}{5V_0^2}\vec{e}'\cdot\vec{e}' - \vec{e}'\vec{e}'\cdot\vec{q}\right] = -\frac{1}{5}\gamma\frac{\vec{q}}{p},$$
$$\underset{\to}{D}_T = 0. \tag{13}$$

4. Thus, the Fokker–Planck equation for the distribution function of Brownian particles in an inhomogeneous gas with (3), (4), (10)-(13) taken into account is given by

$$\frac{\partial F}{\partial t} + \vec{v}\cdot\frac{\partial F}{\partial\vec{r}} + \vec{K}\cdot\frac{\partial F}{\partial\vec{v}} = \frac{\partial}{\partial\vec{v}}\cdot\left[\underset{\to}{\Gamma}\cdot\vec{v}F + \underset{\to}{D}_v\cdot\frac{\partial F}{\partial\vec{v}}\right]. \tag{14}$$

In the above

$$\vec{K} = \vec{E} + \vec{u}\cdot\underset{\to}{\Gamma} + \frac{1}{5}\gamma\frac{\vec{q}}{p},\quad \underset{\to}{\Gamma} = \gamma\left[\underset{\to}{I} - \frac{1}{5}\frac{\underset{\to}{\tau}}{p}\right],\quad \underset{\to}{D}_v = D_v\left[\underset{\to}{I} - \frac{3}{5}\frac{\underset{\to}{\tau}}{p}\right]. \tag{15}$$

In contrast to Brownian motion in a local equilibrium medium, the tensor of dynamic friction $\underset{\to}{\Gamma}$ and the diffusion coefficient $\underset{\to}{D}_V$ depend on the stress tensor in the gas. It is not difficult to find that the Langevin stochastic equation can in this case be given in the form

$$\frac{d\vec{r}(t)}{dt} = \vec{v}(t), \tag{16}$$

$$\frac{d\vec{v}(t)}{dt} = \vec{K}[\vec{r}(t)] - \underset{\to}{\Gamma}[\vec{r}(t)]\cdot\vec{v}(t) + \underset{\to}{B}[\vec{r}(t)]\cdot\vec{F}(t), \tag{17}$$

where $\underset{\to}{B}\cdot\underset{\to}{B} = \underset{\to}{D}_V$, $\vec{F}(t)$ is the Gaussian random process with $\langle\vec{r}(t)\rangle = 0$ and $\langle\vec{F}(t)\vec{F}(t')\rangle = 2\underset{\to}{I}\delta(t-t')$.

The dependence of the coefficients in (17) on $\vec{r}$ makes the solving of (16) and (17) much more difficult in the general case. The random process $(\vec{r}(t), \vec{v}(t))$ appears as a two-component Markov diffusion process. However, bearing in mind that during the relaxation time of the velocity of Brownian particles to local equilibrium state in accordance with the state of the external medium, the distribution in the coordinate space changes only slightly, one can consider Eq. (17) independently of (16) taking into account that the dependence of the coefficients on $\vec{r}$ in (17) is only parametric. This corresponds to a change in $\vec{K}[\vec{r}(t)]$, $\underset{\to}{\Gamma}[\vec{r}(t)]$, and $\underset{\to}{B}[\vec{r}(t)]$ being ignored during times of the order of duration of relaxation of particle velocity. The solution of Eq. (17) can then be represented by

$$\vec{v}(t) = \underset{\to}{\Omega}(t)\cdot[\vec{v}(0) - \underset{\to}{\Gamma}^{-1}\cdot\vec{K}] + \underset{\to}{\Gamma}^{-1}\cdot\vec{K} + \underset{\to}{\Omega}(t)\cdot\int_0^t \underset{\to}{\Omega}^{-1}(s)\cdot\underset{\to}{B}\cdot\vec{F}(s)\,ds, \tag{18}$$

where $\Omega(t) = \exp(-\underset{\to}{\Gamma}t)$, and the particle velocity is approximated by a Gaussian process. It follows from (18) that

$$\langle\vec{v}(t)\rangle = \underset{\to}{\Omega}(t)\cdot[\langle\vec{v}(0)\rangle - \underset{\to}{\Gamma}^{-1}\cdot\vec{K}] + \underset{\to}{\Gamma}^{-1}\cdot\vec{K},$$
$$\langle[\vec{v}(t) - \vec{v}(0)]^2\rangle = \underset{\to}{\Gamma}^{-1}\cdot\underset{\to}{D}_v + \underset{\to}{\Omega}(t)\cdot[\langle\vec{v}(0)\vec{v}(0)\rangle - \underset{\to}{\Gamma}^{-1}\cdot\underset{\to}{D}_v]\cdot\underset{\to}{\Omega}(t) = \underset{\to}{d}^{-1}(t).$$

Therefore, the probability density of the transfer $\vec{v}(t)$ is given by

$$F(t, \vec{v}|\vec{v}') = \left[\frac{\det\underset{\to}{d}(t)}{(2\pi)^3}\right]\exp\left\{-\frac{1}{2}[\vec{v} - \langle\vec{v}(t)\rangle]\cdot\underset{\to}{d}(t)\cdot[\vec{v} - \langle\vec{v}(t)\rangle]\right\}. \tag{19}$$

For $t \to \infty$ one has $\langle \vec{v}(t) \rangle \to \underset{\to}{\Gamma}^{-1} \cdot \vec{K}$, $\underset{\to}{d}(t) \to \underset{\to}{\Gamma} \cdot \underset{\to}{D}_V^{-1}$ and (19) yields the Maxwell velocity distribution of Brownian particles in an inhomogeneous medium

$$F_0(\vec{v}) = \left[\frac{\det \underset{\to}{\Gamma} \cdot \underset{\to}{D}_v^{-1}}{(2\pi)^3} \right]^{\frac{1}{2}} \exp \left[-\frac{1}{2} (\vec{v} - \underset{\to}{\Gamma}^{-1} \cdot \vec{K}) \cdot \underset{\to}{\Gamma} \cdot \underset{\to}{D}_v^{-1} \cdot (\vec{v} - \underset{\to}{\Gamma}^{-1} \cdot \vec{K}) \right]. \tag{20}$$

Since $\mathrm{Tr}\, \underset{\to}{\tau} = 0$ it follows from (18) that the correlation time $\tau_V = [\langle \vec{v}(0)\vec{v}(0) \rangle]^{-1} : \int_0^\infty \langle \vec{v}(t)\vec{v}(0) \rangle dt = \mathrm{Tr}\, \underset{\to}{\Gamma}^{-1}$ $= \gamma^{-1}$ is identical with the duration of correlation for a particle velocity in a local equilibrium medium.

5. The distribution function for particles in the coordinate space is now introduced by

$$P(t, \vec{r}) = \int d\vec{v} F(t, \vec{r}, \vec{v}). \tag{21}$$

The asymptotic behavior of $P(t, \vec{r})$ for $t \gg \tau_V$ (for approximations which are related by the expression (18) to the approximation of velocity) can be analyzed using (14) with the aid of the Chapman–Enskog method or the Kramers method. However, by using the stochastic interpretation of particle motion (with the same approximations) one is able to analyze the behavior of $P(t, \vec{r})$ for $t \lessgtr \tau_V$, as well, that is, as taking into account the effect of inertia of Brownian particles in their motion in the coordinate space.

For simplicity, the case is considered in which $\vec{v}(0)$ is given by the distribution (20). In this case by regarding $\vec{v}(t)$ as a Gaussian random process which parametrically depends on $\vec{r}$, one introduces the function $\mu(t, \vec{r}) = P_0(\vec{r} - \vec{r}^*(t))$, $\vec{r}^*(t)$ which is a solution of (16) and (18), $P_0(\vec{r}) = P(0, \vec{r})$. Then $P(t, \vec{r}) = \langle \mu(t, \vec{r}) \rangle$. It follows from (16) that $\mu(t, \vec{r})$ is a solution of the equation

$$\frac{\partial \mu}{\partial t} = (L_0 + L_1)\, \mu. \tag{22}$$

In the above

$$L_0 = -\nabla \cdot \underset{\to}{\Gamma}^{-1} \cdot \vec{K}, \quad L_1 = -\nabla \cdot \vec{w}(t, \vec{r}), \tag{23}$$

$$\vec{w}(t, \vec{r}) = \underset{\to}{\Omega}(t) \cdot [\vec{v}(0) - \underset{\to}{\Gamma}^{-1} \cdot \vec{K}] + \underset{\to}{\Omega}(t) \cdot \int_0^t \underset{\to}{\Omega}^{-1}(s) \cdot \underset{\to}{B} \cdot \vec{F}(s)\, ds. \tag{24}$$

Averaging the formal solution of (22) one obtains

$$P(t, \vec{r}) = \left\langle T \exp \left[\int_0^t dt' (L_0 + L_1) \right] \right\rangle P_0(\vec{r})$$

or

$$P(t, \vec{r}) = U(t) \left\langle \exp \left[\int_0^t dt' L_1^*(t') \right] \right\rangle P_0(\vec{r}), \tag{25}$$

where

$$U(t) = \exp \left[\int_0^t dt' L_0(t') \right]; \quad L_1^*(t) = U^{-1}(t)\, L_1(t)\, U(t). \tag{26}$$

Employing the definition of semiinvariants of correlation functions of a random process [8] one can represent (25) as

$$P(t, \vec{r}) = U(t) \exp \left[\int_0^t dt' G(t') \right] P_0(\vec{r}), \tag{27}$$

where

$$G(t) = \frac{\partial}{\partial t} \left\langle \exp \left[\int_0^t dt' L_1^*(t') \right] - 1 \right\rangle^c, \tag{28}$$

$\langle L_1^*(t_1) \ldots L_1^*(t_n) \rangle^c$ being semiinvariants of the correlation functions $L_1^*(t)$. The diffusion equation is obtained by differentiating (27) with respect to time:

$$\frac{\partial P(t, \vec{r})}{\partial t} = [L_0 + U(t)\, G(t)\, U^{-1}(t)]\, P(t, \vec{r}). \tag{29}$$

The adoption of the assumption on the coefficients in (17) results in considerable simplification of calculations of the explicit form of the operator $G(t)$. This assumption is essentially equivalent to the operators $U(t)$ and $L_1(t)$ being commutative in the expressions for the correlation functions $L_1^*(t)$ in (28).

Since $\vec{w}(t, \vec{r})$ is a Gaussian process, only the first term does not vanish of the expansion of the exponential operator in (28). Having computed the correlation function $\vec{w}(t, \vec{r})$ and integrated with respect to time, the following expression is found:

$$U(t)G(t)U^{-1}(t) = \frac{1}{2}\nabla\cdot[\underset{\to}{I} - \underset{\to}{\Omega}(t)]\cdot\underset{\to}{\Gamma}^{-1}\cdot(\nabla\cdot\underset{\to}{\Gamma}\cdot\underset{\to}{D}_R) + \nabla\cdot\underset{\to}{D}_R(t)\cdot\nabla, \tag{30}$$

where

$$\underset{\to}{D}_R(t) = \underset{\to}{D}_R[\underset{\to}{I} - \underset{\to}{\Omega}(t)], \quad \underset{\to}{D}_R = \underset{\to}{\Gamma}^{-2}\cdot\underset{\to}{D}_v. \tag{31}$$

The first term in (30) represents additional contribution to the pulling-down determined by the inhomogeneity properties of the surrounding medium, the second term describing the diffusion of Brownian particles in the coordinate space. Using (30) and (23) one can represent Eq. (29) as

$$\frac{\partial P(t, \vec{r})}{\partial t} + \nabla\cdot\left[\underset{\to}{\Gamma}^{-1}\cdot\vec{K} - \frac{1}{2}[\underset{\to}{I} - \underset{\to}{\Omega}(t)]\cdot\underset{\to}{\Gamma}^{-1}\cdot(\nabla\cdot\underset{\to}{\Gamma}\cdot\underset{\to}{D}_R)\right]P(t, \vec{r}) = \nabla\cdot\underset{\to}{D}_R(t)\cdot\nabla P(t, \vec{r}). \tag{32}$$

Unlike the asymptotic equation of Kramers–Smoluchowski, Eq. (32) describes the diffusion process also for $t \lesssim \tau_V$. In particular, the quantity which describes the propagation rate of perturbations, $(d/dt) \cdot \sqrt{\mathrm{Tr}\{[\underset{\to}{\Gamma}t + \underset{\to}{\Omega}(t) - \underset{\to}{I}]\cdot\underset{\to}{\Gamma}^{-1}\cdot\underset{\to}{D}_V\}}$, obtained from (32) remains finite for $t \to 0$, since it is equal to the average velocity of thermal motion of the particles. For $t < \tau_V$ in (32) the inertial effects of the particle motion are essential; they are determined by the time dependence of the coefficient $\underset{\to}{D}_R(t)$ and are related to the effect of velocity fluctuations of a Brownian particle on its motion in the coordinate space. For $t \gg \tau_V$ these effects become inessential, and by ignoring the exponentially damped terms in the expressions for the coefficients in (32) one obtains an asymptotic equation for Brownian diffusion in an inhomogeneous gas

$$\frac{\partial P(t, \vec{r})}{\partial t} + \nabla\cdot\vec{V}P(t, \vec{r}) = \nabla\cdot\underset{\to}{D}_R\cdot\nabla P(t, \vec{r}), \tag{33}$$

where

$$\vec{V} = \underset{\to}{\Gamma}^{-1}\cdot\left[\vec{K} - \frac{1}{2}(\nabla\cdot\underset{\to}{\Gamma}\cdot\underset{\to}{D}_R)\right]. \tag{34}$$

The expressions for $\vec{V}$ and $\underset{\to}{D}_R$ are now analyzed in their linear approximation in the thermo- and stress-bearing effects. Expanding (31) and (34) into power series in $\underset{\to}{\tau}/p$ and retaining only the linear terms one obtains

$$\underset{\to}{D}_R = D_R\left[\underset{\to}{I} - \frac{1}{5}\frac{\underset{\to}{\tau}}{p}\right], \quad \vec{V} = \gamma^{-1}\vec{E}\cdot\left[\underset{\to}{I} - \frac{1}{5}\frac{\underset{\to}{\tau}}{p}\right] + \frac{\vec{q}}{5p} + \nabla\cdot\underset{\to}{D}_R + \vec{u}, \tag{35}$$

which are identical with the results in [4] where $D_R = \gamma^{-2}D_V$ is the Einstein diffusion coefficient.

6. To illustrate the obtained results a simple example is considered of Brownian diffusion of particles in a gas Poiseuille flow; the effect is studied of tangential stresses between parallel plates $y = \pm h/2$ on the stationary distribution of particles. We only retain the linear in τ expressions (35). By adopting the 0X axis in the $\vec{u}$ direction one has $u_x = (3/2)\bar{u}_x(1-4y^2/h^2)$, $u_y = u_z = 0$, $\tau_{xy} = \tau_{yx} = -3\mu\bar{u}_x 4y/h^2$, $\tau_{xx} = \tau_{yy} = \tau_{zz} = \tau_{xz} = \tau_{yz} = 0$. Let $E_x = -E = \mathrm{const}$, $E_y = E_z = 0$. Then it follows from (35) that

$$\begin{gathered} D_{Rxx} = D_{Ryy} = D_R, \quad D_{Rxy} = D_{Ryx} = \frac{3}{5}\frac{D_R}{p}\frac{4}{h^2}\mu\bar{u}_x y, \\ V_x = -\gamma^{-1}E + \frac{3}{2}\bar{u}_x\left(1 - 4\frac{y^2}{h^2}\right) - \frac{3}{5}\frac{D_R}{p}\frac{4}{h^2}\mu\bar{u}_x, \\ V_y = \frac{3}{5}\frac{\gamma^{-1}E}{p}\frac{4}{h^2}\mu\bar{u}_x y. \end{gathered} \tag{36}$$

The stationary diffusion equation then becomes

$$\frac{d}{dy}byP(y) = D_R\frac{d^2P(y)}{dy^2}, \tag{37}$$

where $b = (3/5)\gamma^{-1}E(4/h^2)(\mu\bar{u}_x/pD_R)$. The solution of (37) for the condition $dP/dy|_{\pm h/2} = 0$ can be represented as

$$P(y) = \bar{n}\frac{2\varkappa_0}{\sqrt{\pi}}v^{-1}(\varkappa_0, 0)\exp[\varkappa^2 - \varkappa_0^2]. \tag{38}$$

In the above

$$\varkappa = y\sqrt{b/2};\ \varkappa_0 = \frac{h}{2}\sqrt{b/2};\ v(\varkappa_0, 0) = \frac{2}{\sqrt{\pi}} e^{-\varkappa_0^2} \int_0^{\varkappa_0} dt e^{t^2};$$

$\bar{n}$ is the mean particle density in the passage section. If in (38) the effect is ignored of tangential stresses in flow on the particle distribution ($b \to 0$) then the obvious result $P = \bar{n}$ is obtained. Thus, the stress-bearing effect on Brownian diffusion results in an inhomogeneous distribution of particles in the passage section with a higher concentration nearer the walls. It is noted that in the case of the directions of the external force and of gas velocity being the same ($|b < 0|$), (38) describes an inhomogeneous distribution with lowered particle concentration near the walls. Since (8) is valid if $|\max \tau/p| < 1$ and $\varkappa_0^2 \sim Ehm/kT \cdot |\max \tau/p|$, therefore the inequality $\varkappa_0^2 < 1$ depends on $Ehm/(kT)$ and it is virtually always valid. Therefore, the inhomogeneity of the stationary particle distribution in the gas flow is small. For $b < 1$ the expression (38) can be simplified:

$$P = \bar{n}\left[1 - \frac{b}{2}\left(\frac{1}{3}\frac{h^2}{4} - y^2\right)\right]. \tag{39}$$

A more essential effect of stress-bearing is on the integral flow j_x. The latter is determined by the inhomogeneous distribution (39) on the one hand, and on the other hand by the supplementary drift in the opposite direction to the gas flow which is described by the last term of V_x in (36). By using (36) and (39) one obtains

$$j_x = \int_{-h/2}^{h/2} dy P V_x = \bar{n}h\left[\bar{u}_x - \gamma^{-1}E - \frac{1}{5}\bar{u}_x\left(\frac{1}{3}b\frac{h^2}{4} + 3\frac{D_R 4\mu}{ph^2}\right)\right]. \tag{40}$$

In (40) the stress-bearing effect determines a nonlinear dependence of j_x on the average gas velocity $\bar{u}_x$. It is noted that the validity of (40) is bounded by the values of $\bar{u}_x$ which can be found from the conditions $b < 1$, $|\max \tau/p| < 1$.

NOTATION

$\vec{v}$ and $\vec{v}_0$ are the particle and gas molecule velocity;
σ is the dissipation cross section;
$\vec{u}$ is the average gas flow rate;
T is the gas temperature;
$\underline{\tau}$ is the stress tensor;
$\vec{q}$ is the heat flux;
$V_0^2 = kT/m_0$;
$\underline{I}$ is the identity tensor;
$\vec{\tau}_V$ is the correlation time of particle velocity;
μ is the viscosity;
p is the hydrostatic pressure.

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